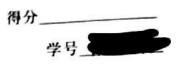
中国科学技术大学 2019-2020学年第一学期考试试卷

考试科目: <u>同调代数</u> 学生所在系:





- Let C, D be additive categories.
- (1) Let $X, Y \in \mathcal{C}$. Then the following are equivalent:
 - (i) $(X \oplus Y, \iota_1 : X \longrightarrow X \oplus Y, \iota_2 : Y \longrightarrow X \oplus Y)$ is a coproduct.
 - (ii) There exist morphisms $p_1: X \oplus Y \longrightarrow X$ and $p_2: X \oplus Y \longrightarrow Y$ such that

$$p_1\iota_1=1_X,\ p_2\iota_2=1_Y,\ \iota_1p_1+\iota_2p_2=1_{X\oplus Y}.$$

- (2) Let F: C → D be a (not necessarily additive) functor which admits a left (or right) adjoint. Then F is additive.
- Let C be a category with zero object. Prove that:
 - (1) Coequalizers and cokernels are direct limits.
 - (2) Assume C has finite coproduct, then C admits coequalizers iff C admits pushouts.
- Denote the category of left R-modules by RMod. Prove that:
- (1) Let $0 \longrightarrow A_i \longrightarrow B_i \longrightarrow C_i \longrightarrow 0$ be a family of short exact sequences in ${}_{R}\mathbf{Mod}$, then $0 \longrightarrow \Pi_i A_i \longrightarrow \Pi_i B_i \longrightarrow \Pi_i C_i \longrightarrow 0$ is exact.
- (2) Let I be a direct set, and $\{A_i, \alpha_j^i\}, \{B_i, \beta_j^i\}, \{C_i, \gamma_j^i\}$ be direct systems in R**Mod** over I. Assume that $r: \{A_i, \alpha_j^i\} \longrightarrow \{B_i, \beta_j^i\}$ and $s: \{B_i, \beta_j^i\} \longrightarrow \{C_i, \gamma_j^i\}$ are morphism of direct systems and such that $0 \longrightarrow A_i \xrightarrow{r_i} B_i \xrightarrow{s_i} C_i \longrightarrow 0$ is exact for each $i \in I$. Then there is an exact sequence

$$0 \longrightarrow \varinjlim A_i \longrightarrow \varinjlim B_i \longrightarrow \varinjlim C_i \longrightarrow 0.$$

(3) Keep assumptions as in (2). Prove or disprove the exactness of

$$0 \longrightarrow \varprojlim A_i \longrightarrow \varprojlim B_i \longrightarrow \varprojlim C_i.$$

(4) Give a counter-example to show that (2) is NOT true if I is NOT direct.

- 4. Compute $H_i(\mathbb{Z}_m \times \mathbb{Z}_n, \mathbb{C}^{\times})$ and $H^i(\mathbb{Z}_m \times \mathbb{Z}_n, \mathbb{C}^{\times})$ for i = 0, 1, 2, where \mathbb{C}^{\times} is the multiplicative group of the complex number field.
- 5. Let R be a P.I.D., B a left R-module, and (C, d) be a complex of free left R-module. Then there exist exact sequences

$$0 \longrightarrow \operatorname{Ext}_{R}^{1}\left(\operatorname{H}_{n-1}(\mathbf{C}), B\right) \xrightarrow{\lambda_{n}} \operatorname{H}^{n}\left(\operatorname{Hom}_{R}(\mathbf{C}, B)\right) \xrightarrow{\mu_{n}} \operatorname{Hom}_{R}\left(\operatorname{H}_{n}(\mathbf{C}), B\right) \longrightarrow 0, n \in \mathbf{Z}.$$

- Let f: R → R* be a ring homomorphism.
- (1) The induced functor $f^*: R^*Mod \longrightarrow RMod$ is exact and admits left adjoint and right adjoint.
 - (2) If R* is a projective R-module, then

$$\operatorname{Ext}_{R^{\bullet}}^{n}(A^{\bullet}, \operatorname{Hom}_{R}(R^{\bullet}, B)) \xrightarrow{\cong} \operatorname{Ext}_{R}^{n}(f^{\bullet}A^{\bullet}, B).$$

(3) If R* is a flat R-module, then

$$\operatorname{Ext}_{R^*}^n(R^*\otimes_R A, B^*) \xrightarrow{\cong} \operatorname{Ext}_R^n(A, f^*B^*).$$

- 7. Let K be a field and A be a K-algebra. The n-th Hochschild cohomology group of A with coefficient in A is defined as $HH^n(A, A) := \operatorname{Ext}_{A^e}^n(A, A)$, where $A^e := A \otimes_{\mathbb{K}} A^{op}$.
 - Give all indecomposable projective modules of A^e.
 - (2) Compute $HH^0(A, A) = Z(A)$ and $HH^1(A, A)$.
- (2) Compute III (A, A) = E(A) (3) Let A be a matrix subalgebra of Mat₃(K) whose elements with the form $\begin{pmatrix} * & * & * \\ 0 & * & * \\ 0 & 0 & * \end{pmatrix}$ Compute $HH^n(A, A)$, $n = 0, 1, 2, \cdots$.
- 8. If every finitely generated submodule of a left R-module M is flat. Show that M is flat.