1.given a dihedral group $D_7 =< \sigma, \tau | \sigma^7 = \tau^2 = e, \tau\sigma\tau = \sigma^{-1} >$.

(1)prove that D_7 is the only non-abelian group of order 14.

(2)construct a injective group homomorphism $D_7 \hookrightarrow S_7$

2.let α be an algebraic number with monic minimal polynomial $f(x)$ over $\mathbb{Q}$.

(1)given a monic polynomial $h(x) \in \mathbb{Q}[x]$ s.t. $h(\alpha) = 0$,prove that $f(x)|h(x)$

(2)prove:$f(x) \in \mathbb{Z}[x]$ iff $\exists$ monic polynomial $h(x) \in \mathbb{Z}[x]$ s.t.$h(\alpha) = 0$

3.let $\mathbb{F}_q$ be the finite field of order $q = p^r$,where r is a positive integer,p is prime integer.calculate the order of group $GL_n(\mathbb{F}_q)$,then find a Sylow p-subgroup.

4.let R be a commutative ring.given a $R-$module commutative diagram with exact rows as follow.

$$\begin{array}{ccccccc} & & M' & \longrightarrow & M & \longrightarrow & M'' & \longrightarrow & 0 \\ & & \big\downarrow{\scriptstyle f} & & \big\downarrow{\scriptstyle g} & & \big\downarrow{\scriptstyle h} & & \\ 0 & \longrightarrow & N' & \longrightarrow & N & \longrightarrow & N'' & & \end{array}$$

prove:

(1)suppose f, h is injective ,then g is injective.

(2)suppose f, h is surjective ,then g is surjective.

(3)suppose $0 \to M' \to M$ and $N \to N'' \to 0$ are exact sequences.prove that any two of f, g, h are isomorphic implies the rest one is also isomorphic

5.prove:$Q(\sqrt{2} + \sqrt{3})/Q$ is Galois extension.calculate its Galois group.

6.given a PID R, finitely generated $R-$module M, N such that $M \oplus M \cong N \oplus N$. prove:$M \cong N$.

7.let G be finitely generated abelian group with n generators, H is a subgroup of G. prove: H is finitely generated abelian group with $\leq n$ generators.(do not use theorem about classification of finitely generated abelian group.) 8.construct examples as required:

(1)non-zero R-module M and N s.t. $M \otimes_R N = 0$.

(2)field extension E/F is normal extension ,but not Galois.