

1. consider formal power series ring over \mathbb{C} , $R = \mathbb{C}[[x]] = \{a_0 + a_1x + a_2x^2 + \dots \mid a_i \in \mathbb{C}\}$

(1) prove: R is Noetherian ring.

(2) find all finitely generated indecomposable R -module.

(3) find all double-sided ideals of $M_2(R)$.

(4) find all finitely generated indecomposable left $M_2(R)$ -module, and the endomorphism ring of these modules.

2. take abelian group as \mathbb{Z} -module. consider abelian group $G = (\mathbb{Z}/3\mathbb{Z}) \oplus \mathbb{Z}$.

(1) find all subgroup of G .

(2) find all quotient group of G in the form of direct sum of indecomposable \mathbb{Z} -module.

(3) find all subgroup A of G s.t. \exists subgroup B of G , $G = A \oplus B$

(4) describe $\text{Aut}(G)$.

3. construct \mathbb{C} -algebra isomorphism concretely:

$$\mathbb{C}[S_3] \xrightarrow{\sim} \mathbb{C} \times \mathbb{C} \times M_2(\mathbb{C})$$